

# Active Vibration Control of Piezoelectric Stewart Platform Based on Fuzzy Control

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## Abstract

This paper presents a simulation study aimed at investigating the potential of using fuzzy controller for active vibration control in a piezoelectric Stewart platform. The focus of the study is to assess, through simulation, the control authority of the piezo stack actuators for effectively controlling the Stewart platform vibration. Dynamic analysis of the structure is performed using Matlab/Simulink. Each leg of the active interface consists of a linear piezo stack actuator, a collocated force sensor and flexible tips for the connections with the two end plates. The piezoelectric stack is modeled as a bar element and the electro-mechanical coupling property is simulated. The open loop and closed loop dynamic response simulations are done in Matlab/Simulink. The closed loop implementation is aimed at achieving maximum damping of the six rigid body modes of the system, through six local fuzzy force feedback controllers. Simulations are carried out in Matlab/Simulink to characterize the effect of control on the overall response of the closed loop control to white noise disturbance forces, with the constraint on the stack actuator voltage to be within a specified bound. Using fuzzy controllers shows improvement in the damping of the structure.

## Keywords

*Active Vibration Control; Fuzzy Controller; Piezoelectric Stewart Platform.*

## Introduction

Future space telescope will need much more improved angular resolution than the Hubble Space Telescope. This required resolution can only be achieved in two ways, by much larger telescopes or interferometric devices, where the signals of the several independent smaller telescopes are combined together to create the global resolution. Interferometric devices cost less than a larger telescope solution. Therefore, it is logical to follow this procedure in the upcoming future. Future space interferometers consist of various telescopes mounted on a very large truss which will be subjected to static and dynamic disturbances such as thermal loads, gravity loads, attitude control, etc. These disturbances in the form of vibrations can reduce the

resolution of the system. Therefore, vibration isolation is essential. One kind of the vibration isolation method is passive vibration isolation which can effectively isolate high frequency vibration. It has been proved to be inappropriate to isolate low frequency vibration. Also, passive vibration isolation can hardly handle the uncertainties of the system. The other kind is active vibration isolation techniques with the feedback control which can overcome the drawbacks of passive vibration isolation mentioned above. It can improve the vibration isolation performance in the low frequency vibration ranges but it requires sensors, actuators, and processors. With the development of smart sensors and actuators, active vibration is becoming an attractive solution to vibration isolation. As mentioned before, it is important to create a vibration-free environment for the space structures. This is where parallel mechanisms such as Stewart platform mechanism come out as an ideal candidate. A Stewart platform mechanism is a six DOF parallel manipulator consisting of a fixed base plate and an upper moving plate, connected together by six variable link lengths (Stewart, 1965). Such a platform has been proven to have high positioning accuracy while maintaining high force-to-weight ratio over conventional serial manipulator. It can be used as active mount for quiet components, isolation mount for a disturbance source, active structural element of trusses for vibration control. According to the stiffness of the legs, Stewart platforms can be classified into two main categories; stiff and soft (Hanieh et al., 2002a). The stiff platform uses piezoelectric or magnetostrictive legs, while the soft platform in which each leg acts with a voice coil actuator can provide far more actuation stroke than stiff design (1000  $\mu\text{m}$  or more). The main focus of the paper is on the stiff piezoelectric Stewart platform. The potential applications for the piezoelectric Stewart platform are precision pointing device, vibration isolation, active damping interface and a possible combination with pointing. The design, manufacturing and applications

of piezoelectric Stewart platforms have been discussed in several researches (Hanieh et al., 2001; Hanieh et al., 2002a; Hanieh et al., 2002b; Loix et al., 2002; Wang, 2009). However, there has been a great number of control design approaches proposed in recent years for the Stewart platform such as robust PD controller (Kang et al., 1996), sliding mode control (Kim et al., 1998), adaptive control (Gengand Haynes, 1994), etc., in these researches (Hanieh et al., 2001; Hanieh et al., 2002a; Hanieh et al., 2002b; Loix et al., 2002; Wang, 2009), only the integral force feedback and adaptive controller have been used for the active vibration control of the piezoelectric Stewart platform. In the past decade, fuzzy theory has been used in many engineering applications. A rapid growth in the use of fuzzy logic in a wide variety of consumer products and industrial systems can be seen. To the best knowledge of the authors, there have been no attempts to apply fuzzy controller for the active vibration control of the piezoelectric Stewart platform. In this paper, a fuzzy controller has been used to introduce damping in the mechanical system attached to it while remaining stiff. The active interface consists of a six-degree of freedom Stewart platform, a standard hexapod with a cubic architecture. Each leg of the active interface is made of a linear piezoelectric actuator, a collocated force sensor and flexible tips for the connection with the two end plates. The control architecture is based on six local/decentralized Fuzzy Force Feedback controllers. By providing the legs with strain or elongation sensors, this active interface can also be used as an interface at low frequency (i.e. for machine tools), a six-degree of freedom positioning device for space applications as well as a micro vibration isolator.

## Modeling

The proposed Stewart platform is based on the cubic configuration which was invented by the Intelligent Automation Inc (IAI) (Gengand Haynes, 1994). The cubic configuration has several characteristics such as: uniform stiffness, uniform control capability in all directions and concise kinematics and dynamic analysis. The nominal configuration can be obtained by cutting a cube of two planes as shown in FIG. 1. The two triangular planes are the base and the mobile platforms of the Stewart platform. The edges of the cube connecting the two plates are the six legs of the hexapod. Vibration isolation utilizing the cubic configuration of Stewart platform has been investigated by Gengand Haynes (1994), Spanos et al.

(1995), and Thayer et al. (2002). The active legs of the system are made of a piezo stack actuator and a collocated force sensor. A voltage is produced which is proportional to the measured force of the force sensor. Then, this signal is used to generate the control signal according to an appropriate control algorithm. Next, the control signal is fed to a high voltage amplifier which drives the actuator. The mentioned algorithm will be implemented in Matlab/Simulink in the next section. When a voltage  $V$  is applied to a linear piezoelectric actuator, it creates an expansion as  $\delta$  :

$$\delta = d_{33}nV = g_a V \quad (1)$$

Where  $d_{33}$  is the piezoelectric coefficient,  $n$  is the number of piezoelectric ceramic layers in the actuator and  $g_a$  is the actuator gain. The effect of actuator on the Stewart platform can be represented by equivalent piezoelectric loads acting on the structure. The piezoelectric load which is applied axially to both ends of the active strut is equal to:

$$f = k\delta \quad (2)$$

Where  $k$  is the axial stiffness of the active strut and  $\delta$  is the unconstrained piezoelectric expansion. Due to slight displacements of the piezo actuators, the nonlinear terms such as centripetal and coriolis forces can be neglected. Therefore, the motion equation of the Stewart platform without these terms and damping excited by six actuators is:

$$M\ddot{X} + KX = F + Bf = F + Bk\delta \quad (3)$$

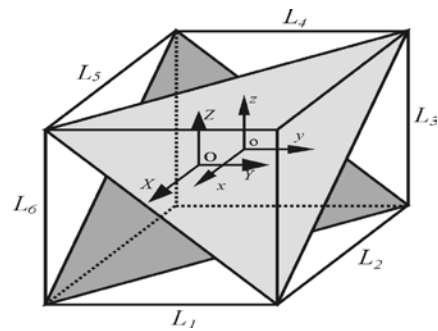


FIG. 1 CUBIC CONFIGURATION OF STEWART PLATFORM

Where  $M$  is the inertia matrix of the Stewart platform,  $K$  is the stiffness matrix and the vector  $F$  is the disturbance forces and moments acting on the payload platform  $F = [F_x \ F_y \ F_z \ M_x \ M_y \ M_z]^T$ .  $X = [x \ y \ z \ \psi \ \theta \ \phi]^T$  is a vector containing the translational

displacements  $x, y, z$ , and rotational displacements  $\psi, \theta, \phi$  of the end-effector center about the fixed axes of  $x, y, z$  respectively,  $B$  is the influence matrix of the active struts in the fixed coordinate system known as force Jacobian matrix,  $f = [f_1, \dots, f_6]^T$  is the vector of actuator forces defined by (2), and  $\delta = (\delta_1, \dots, \delta_6)^T$  is the vector of the six unconstrained displacements of the piezoelectric actuators. The inertia matrix of the Stewart platform can be expressed by:

$$M = \begin{bmatrix} m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & 0_{3 \times 3} \\ 0_{3 \times 3} & P^T \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} P \end{bmatrix} \quad (4)$$

$$P = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\psi & s\psi c\theta \\ 0 & -s\psi & c\psi c\theta \end{bmatrix}$$

Where  $m$  is the mass of the end-effector;  $I_x, I_y, I_z$  are the moments of inertia of the moving platform expressed in the moving coordinate  $\{P\}$ . The end-effector is deflected away from its desired position in the presence of the external forces. The overall stiffness matrix  $K_u$  of the Stewart platform can be defined as:

$$K = B \times \text{diag}(k_1, k_2, k_3, k_4, k_5, k_6) \times B^T \quad (5)$$

Where

$$k_i = \frac{A_i E_i}{l_i} \quad i = 1, 2, \dots, 6 \quad (6)$$

In which  $E_i$  is the elasticity module of piezo,  $A_i$  is the cross section area of piezo stack and  $l_i$  is the nominal length of each leg. This matrix is always positive semi-definite and symmetric and because of its dependence on the Jacobian, it is also dependent on the posture of the end-effector. For obtaining the Jacobian matrix  $J$  and the stiffness matrix  $K_u$ , let us consider the vectorial representation of the hexapod as shown in FIG. 2. The cubic configuration of Stewart platform has a fixed platform, a moving platform and six legs each connected to a connection point on the end-effector  $A_i$  and one of the connection points on the fixed triangular platform  $B_i$ , as shown in FIG. 2.  $\{B\}$  is

considered as the inertial reference frame of the fixed platform which coincides with the mass center of the base platform and  $\{P\}$  is considered as the reference frame at the mass center  $C$  of the moving platform.  $r_{base}$  is the radius distance from the origin of  $\{B\}$  to the connection points on the base platform  $B_i$  and  $r_{end}$  is the radius distance from the origin of  $\{P\}$  to the connection points on the moving platform  $A_i$ . This expression can be obtained from FIG. 2:

$$q_i = x_0 + [R]^M p_i - r_i \quad i = 1, 2, 3, \dots, 6 \quad (7)$$

Where  $r_i$  is the position vector of the connection points on the base platform  $B_i$  expressed in  $\{B\}$ ,  $p_i$  is the position vector of the connection points on the moving platform  $A_i$  expressed in  $\{P\}$ ,  $x_0$  is the position vector of point  $C$ ,  $q_i$  is the cable length vector from  $B_i$  to  $A_i$  and  $R$  is the rotation matrix of the moving platform with respect to base platform with three rotation angles  $\psi, \theta, \phi$  about the fixed axes of  $\{B\}$  respectively and can be defined as:

$$[R] = \begin{bmatrix} c\phi c\theta & -s\phi c\psi + c\phi s\theta s\psi & s\phi s\psi + c\phi s\theta c\psi \\ s\phi c\theta & c\phi c\psi + s\phi s\theta s\psi & -c\phi s\psi + s\phi s\theta c\psi \\ -s\theta & c\theta s\psi & c\theta c\psi \end{bmatrix} \quad (8)$$

The cable length of the leg can be defined as:

$$l_i = |q_i| = (q_i^T q_i)^{1/2} \quad (9)$$

The influence matrix  $B$  known as the force Jacobian matrix can be obtained from the virtual work and is written in such way that its column is:

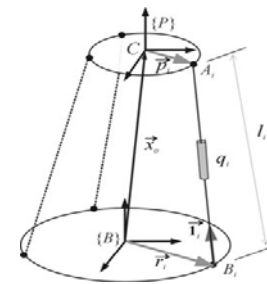


FIG. 2 VECTORIAL REPRESENTATION OF STEWART PLATFORM

$$B_i = \begin{bmatrix} \frac{q_i}{|q_i|} \\ B a_i \times \frac{q_i}{|q_i|} \end{bmatrix}, i = 1, 2, \dots, 6 \quad (10)$$

Where:

$${}^B a_i = [R] p_i \quad (11)$$

As mentioned before, in each leg of the hexapod, there is a force sensor collocated with an actuator. The sensor output equation is:

$$y = k (q - \delta) \quad (12)$$

Where  $y = (y_1, \dots, y_6)^T$  is the six force sensor outputs,  $q = (q_1, \dots, q_6)^T$  is the vector of leg extension from the equilibrium position,  $k$  is the strut stiffness and  $\delta = (\delta_1, \dots, \delta_6)^T$  is the vector of the six unconstrained displacements of the piezoelectric. We know that the relationship between the leg extension and the payload frame displacement can be expressed as  $q = JX = B^T X$  where  $J$  and  $B$  are the velocity and force Jacobian matrices respectively, defined by (10). Therefore we have

$$y = k (B^T X - \delta) \quad (13)$$

### Fuzzy Controller Design

As we know, in a fuzzy-logic controller, the dynamic behavior of a fuzzy system is characterized by a set of linguistic description rules based on expert knowledge usually of the form: IF (a set of conditions are satisfied) THEN (a set of consequence can be inferred). In order to simulate the FFF controller for the structure, the state space of the dynamic equation of the structure (3), the sensor equation (13) and the fuzzy control law are used in Matlab/Simulink to make the closed loop system for piezoelectric Stewart platform. The max-min (Mamdani type) inference is used to generate the best possible conclusion. This type of inference is computationally easy and effective; thus it is appropriate for real-time control applications. The crisp control command is calculated here using the center-of-gravity (COG) defuzzification. For implementation, the fuzzy controller is made of 6 independent sub-fuzzy controllers. The sub-fuzzy controller of each leg contains one input which is the integral of the sensor force  $\int y_i$  and one output which is the actuator force ( $f_i$ ). For all the inputs and outputs, seven membership functions are used. Also, seven control rules are constructed for each subfuzzy controller as shown in TABLE 1. Therefore, the fuzzy controller has a total of 42 fuzzy control rules. FIG. 3 shows the seven triangular membership functions for

all the six inputs ( $\int y$ ) and six outputs ( $f_i$ ). The model of the piezoelectric Stewart platform is shown in FIG. 4.

TABLE 1 RULES FOR THE SIXSUB-FUZZY CONTROLLERS

IF	$\int y_i$	is	THEN	$f_i$	is
	$A_i$			$AA_i$	
	$B_i$			$BB_i$	
	$C_i$			$CC_i$	
	$D_i$			$DD_i$	
	$E_i$			$EE_i$	
	$F_i$			$FF_i$	
	$G_i$			$GG_i$	

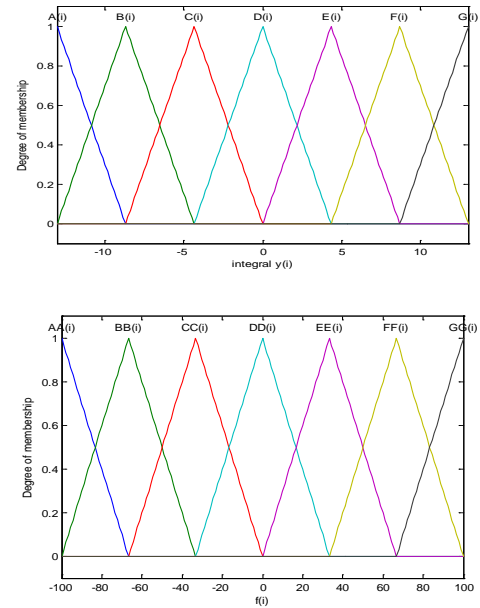


FIG. 3 MEMBERSHIP FUNCTIONS FOR INPUT AND OUTPUT OF SIX SUB-FUZZY CONTROLLERS

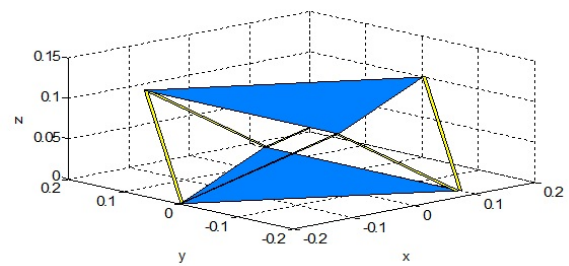


FIG. 4 MODEL OF PIEZOELECTRIC STEWART PLATFORM IN MATLAB/SIMULINK

### Simulation

For simulating the piezoelectric Stewart platform, the state space of the dynamic equation of the structure (3), the sensor equation (13) and the fuzzy control law (Table 1) are used in Matlab/Simulink to make the closed loop system for piezoelectric Stewart platform. It is assumed that three forces and three moments are

applied simultaneously at the center of the end-effector (top platform). With the constraint on the stack actuator voltage to be within 0-20 V, the system responses without and with control for white noise disturbances have been performed. These assumptions are considered as presented in TABLES 2, 3 for the simulation. TABLE 2 shows the geometrical configuration of the Stewart platform. In TABLE 3, the specification of the piezo stack actuators is presented. The six white noise disturbance forces and moments are assumed to be Gaussian distributed random signals with the mean value of zero and the variance value of 25 N<sup>2</sup> and 25 (N.m)<sup>2</sup> for the forces and moments respectively. As shown in FIG. 5, the fuzzy controller made significant improvement in the damping of the structure  $X=[x \ y \ z \ \psi \ \theta \ \phi]^T$ .

TABLE 2 SPECIFICATION OF STEWART PLATFORM

PROPERTY	SYMBOL	VALUE
Mass of end effector (kg )	$m$	1
R end effector (m)	$r_{end}$	0.2
R base(m)	$r_{base}$	0.2
Moments of inertia(kg.m <sup>2</sup> )	$I_x$	0.005
Moments of inertia(kg.m <sup>2</sup> )	$I_y$	0.005
Moments of inertia(kg.m <sup>2</sup> )	$I_z$	0.01
Length of each leg(m)	$l$	0.24

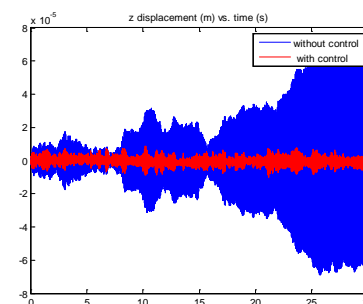
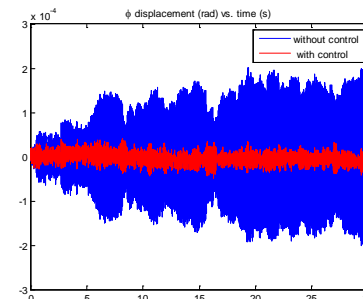
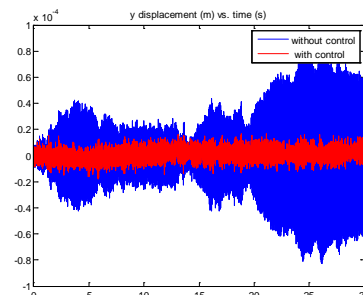
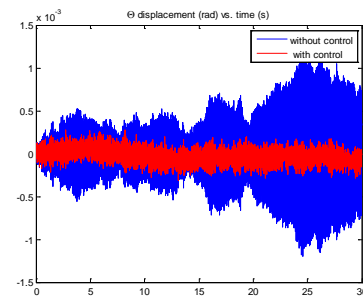
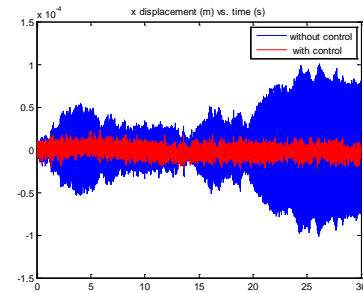
TABLE 3 PIEZOCERAMIC PROPERTIES OF PIEZO STACK ACTUATOR

PROPERTY	SYMBOL	VALUE
Piezo Modulus (Gpa)	$E_p$	70
Piezo Density (kg/m <sup>3</sup> )	$\rho_p$	$7 \times 10^3$
Section area (m <sup>2</sup> )	$A_p$	$3.1 \times 10^{-5}$
Piezo stack length (m)	$L_p$	0.24
Piezo strain coefficient (m/V)	$d_{33}$	$5 \times 10^{-10}$
Thickness of layers (m)	$t$	$1 \times 10^{-4}$

## Conclusions

The focus of the study is to assess, through simulation, the control authority of the piezo stack actuators for effectively controlling the Stewart platform vibration. For this purpose, the dynamic equation of the Stewart platform and the piezo stack actuators and its corresponding force sensors are modeled. Then, six local Fuzzy Force Feedback controllers (FFF) with voltage feedback control law have been used. Simulations are carried out in Matlab/Simulink to characterize the effect of control on the overall response of the closed loop control to white noise

disturbance forces, with the constraint on the stack actuator voltage to be within a specified bound. Using fuzzy controller shows improvement in the damping of the structure.



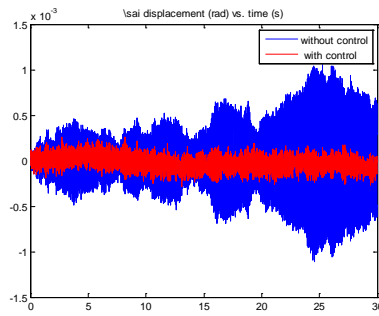


FIG. 5 TRANSLATIONAL AND ROTATIONAL DISPLACEMENTS ( $x, y, z, \psi, \theta, \phi$ )

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